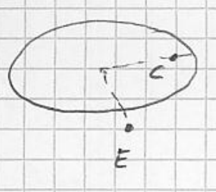
(14.16)



$\theta_i^E = 0$

$\theta_i^C = \frac{\pi}{2}$

$\omega_E = \frac{1}{4}\ \frac{rad}{s}$

$\alpha_C = \frac{1}{100}\ \frac{rad}{s}$

- situation initiale -

$\theta_f^E = \theta_f^C = \theta$

- situation finale -

• Mouvement de l'enfant : $\theta_f^E = \omega_E \cdot \Delta t + \theta_i^E \Rightarrow \theta_f^E = \frac{1}{4}\cdot\Delta t$ (1)

• Mouvement du cheval : $\theta_f^C - \theta_i^C = \frac{1}{2}\alpha_C \cdot \Delta t^2 \Rightarrow \theta_f^C = \frac{1}{2}\frac{1}{100}\Delta t^2 + \frac{\pi}{2}$ (2)

$(1) = (2) \Rightarrow \frac{1}{4}\Delta t = \frac{1}{200}\Delta t^2 + \frac{\pi}{2} \Rightarrow \frac{1}{200}\Delta t^2 - \frac{1}{4}\Delta t + \frac{\pi}{2} = 0 \Rightarrow \Delta t^2 - 50\Delta t + 100\pi = 0$

$\Delta t', '' = \frac{50 \pm \sqrt{2500 - 400\pi}}{2} = \begin{cases}\to 42{,}6\,s \\ \to 7{,}4\,s\end{cases} \nearrow \underline{\underline{7{,}4\,s}}$ le 2ème temps correspond à une autre rencontre côte à côte

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(14.17) . Hypothèse :

a). $\bar{\omega} = \frac{\Delta\theta}{\Delta t}$ (14.1) ; $\omega_f = \alpha\Delta t + \omega_i$ (14.6) ; $\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 + \omega_i\Delta t$ (14.7)

Conclusion :

$\bar{\omega} = \frac{1}{2}(\omega_i + \omega_f)$ (14.8)

démonstration :

$\alpha\Delta t = \omega_f - \omega_i$ (14.6)

de $\bar{\omega} = \frac{\Delta\theta}{\Delta t} \overset{(14.7)}{=} \frac{1}{\Delta t}\left(\frac{1}{2}\alpha(\Delta t)^2 + \omega_i\Delta t\right) = \frac{1}{2}\alpha\Delta t + \omega_i = \frac{1}{2}(\omega_f - \omega_i) + \omega_i = \frac{1}{2}\omega_f + \frac{1}{2}\omega_i$

$= \frac{1}{2}(\omega_i + \omega_f)$ #

b). $\omega(t) = \alpha(t - t_i) + \omega_i$ (14.5)

$$\bar{\omega} = \frac{\int_{t_i}^{t_f}\left[\alpha(t-t_i) + \omega_i\right]dt}{\underbrace{\int_{t_i}^{t_f} dt}_{(t_f - t_i)}} = \frac{1}{(t_f - t_i)}\int_{t_i}^{t_f}(\alpha t\,dt - \alpha t_i\,dt + \omega_i\,dt) = \frac{1}{t_f - t_i}\left[\int_{t_i}^{t_f}\alpha t\,dt - \alpha t_i\int_{t_i}^{t_f}dt + \omega_i\int_{t_i}^{t_f}dt\right]$$

$$= \frac{1}{t_f - t_i}\left[\alpha\frac{1}{2}t^2\Big|_{t_i}^{t_f} - \alpha t_i(t_f - t_i) + \omega_i(t_f - t_i)\right] = \frac{1}{t_f - t_i}\left[\frac{1}{2}\alpha\overbrace{(t_f^2 - t_i^2)}^{(t_f - t_i)(t_f + t_i)} - \alpha t_i(t_f - t_i) + \omega_i(t_f - t_i)\right]$$

$$= \frac{1}{2}\alpha(t_f + t_i) - \alpha t_i + \omega_i = \frac{1}{2}\alpha t_f + \frac{1}{2}\alpha t_i - \alpha t_i + \omega_i = \frac{1}{2}\alpha(t_f - t_i) + \omega_i = \frac{1}{2}\alpha\Delta t + \omega_i$$

$= \frac{1}{2}(\omega_i + \omega_f)$

d'après le résultat de a).