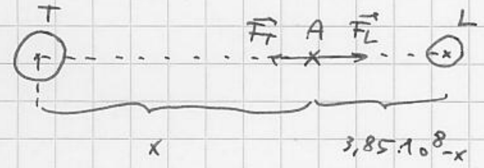
4.23



$x > 0$

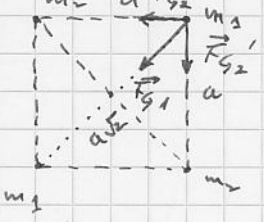
En A : $F_{G_T} = F_{L_T}$

$$\begin{cases} F_{G_T} = G \cdot \dfrac{M_T}{x^2} \cdot m \\ F_{L_T} = G \cdot \dfrac{M_L}{(3{,}85.10^8 - x)^2} \cdot m \end{cases} \Rightarrow \cancel{G} \cdot \cancel{m} \cdot \frac{M_T}{x^2} = \cancel{G} \cdot \cancel{m} \cdot \frac{M_L}{(3{,}85.10^8 - x)^2} \Rightarrow \frac{x^2}{M_T} = \frac{(3{,}85.10^8 - x)^2}{M_L}$$

$$\Rightarrow \frac{M_L}{M_T} = \left(\underbrace{\frac{3{,}85.10^8 - x}{x}}_{>0}\right)^2 \Rightarrow \sqrt{\frac{M_L}{M_T}} = \frac{3{,}85.10^8 - x}{x}$$

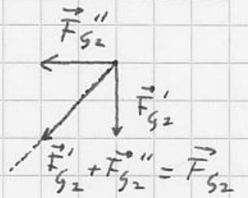
$$\Rightarrow x \cdot \sqrt{\frac{M_L}{M_T}} = 3{,}85.10^8 - x \Rightarrow x\left(1 + \sqrt{\frac{M_L}{M_T}}\right) = 3{,}85.10^8 \Rightarrow x = \frac{3{,}85.10^8}{1 + \sqrt{\frac{M_L}{M_T}}} \cong \underline{\underline{3{,}47.10^8\,m}}$$

4.24



• Sans les masses m_2 : $F_{G_1} = G \cdot \dfrac{m_1^2}{2a^2}$ (seule force)

• Avec les masses m_2 : $F_{G_2} = \sqrt{F_{G_2}'^2 + F_{G_2}''^2} = \sqrt{\left(G \cdot \dfrac{m_1 m_2}{a^2}\right)^2 \cdot 2} = G \cdot \dfrac{m_1 m_2}{a^2} \cdot \sqrt{2}$

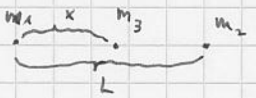


$$F_{Totale\ 1} = F_{G_1} + F_{G_2} = G \cdot \frac{m_1^2}{2a^2} + G \cdot \frac{m_1 m_2}{a^2} \cdot \sqrt{2}$$

$$\frac{F_{Totale\ 1}}{F_{G_1}} = 2 = \frac{\cancel{G} \cdot \frac{m_1^2}{2a^2} + \cancel{G} \cdot \frac{m_1 \cdot m_2}{a^2} \cdot \sqrt{2}}{\cancel{G} \cdot \frac{m_1^2}{2a^2}} = \frac{\frac{1}{2} m_1 + m_2 \cdot \sqrt{2}}{\frac{m_1}{2}} \Rightarrow m_1 = \frac{m_1}{2} + \sqrt{2}\, m_2$$

$$\Rightarrow \frac{m_1}{2} = \sqrt{2}\, m_2 \Rightarrow \underline{\underline{\frac{m_1}{m_2} = 2\sqrt{2}}}$$

4.25



$m_2 = 3 m_1$ et $x ; L - x > 0$

· avant mise en place de m_3 : $F_{①②} = G \cdot \dfrac{m_1 m_2}{L^2}$ identiques

· après mise en place de m_3 :

$$\begin{cases} F_{①} = F_{12} + F_{13} = G \cdot \dfrac{m_1 \cdot m_2}{L^2} + G \cdot \dfrac{m_1 \cdot m_3}{x^2} = 2 \cdot G \dfrac{m_1 m_2}{L^2} & (1) \\ F_{②} = F_{21} + F_{23} = G \cdot \dfrac{m_1 \cdot m_2}{L^2} + G \cdot \dfrac{m_2 \cdot m_3}{(L-x)^2} = 2\, G \dfrac{m_1 m_2}{L^2} & (2) \end{cases}$$

(1) : $G \dfrac{m_1 m_2}{L^2} = G \cdot \dfrac{m_1 m_3}{x^2}$

(2) : $G \cdot \dfrac{m_1 m_2}{L^2} = G \cdot \dfrac{m_2 m_3}{(L-x)^2}$

$$\Rightarrow \cancel{G} \cdot \frac{m_1 \cancel{m_3}}{x^2} = \cancel{G} \cdot \frac{m_2\, \cancel{m_3}}{(L-x)^2} \Rightarrow \frac{m_1}{m_2} = \left(\frac{x}{L-x}\right)^2 = \frac{m_1}{3 m_1} = \frac{1}{3}$$

$$\Rightarrow \frac{x}{L-x} = \frac{\sqrt{3}}{3} \Rightarrow x = \frac{\sqrt{3}}{3} L - \frac{\sqrt{3}}{3} x \Rightarrow x\left(1 + \frac{\sqrt{3}}{3}\right) = \frac{\sqrt{3}}{3} L$$

$$\Rightarrow x = \frac{\sqrt{3}/3\, L}{(3+\sqrt{3})/3} = \underline{\underline{\frac{\sqrt{3}}{3+\sqrt{3}} \cdot L}}$$

· Calcul de m_3 :

de (1) : $\dfrac{m_2}{L^2} = \dfrac{m_3}{x^2} \Rightarrow m_3 = x^2 \cdot \dfrac{m_2}{L^2} = \left(\dfrac{\sqrt{3}}{3+\sqrt{3}}\right)^2 \cdot 3 m_1$

$$m_3 = \frac{3}{4+2\sqrt{3}}\, m_1$$